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Probability distribution of footbridge peak acceleration to single and multiple crossing walkers

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Abstract

The scenarios of a single and of multiple walkers crossing a footbridge are considered by many Standards and design Guidelines for vibration serviceability assessment. Accordingly, this study analyzes the probability distribution of footbridge peak accelerations induced by these two load cases. In particular, single span footbridges with uniform mass distribution are considered, with different values of span length, natural frequencies, and structural damping. Only lateral vibrations are considered, and the load is modeled as a moving sinusoidal force corresponding to the first harmonic. The randomness of the dynamic characteristics of walkers is modeled using probability distributions taken from the literature; so doing a standard probabilistically-modeled population is defined. The footbridge is analyzed by means of modal analysis, considering only the first mode. In the case of multiple crossing walkers, arrival time is modeled as a Poisson distribution and different number of walkers, therefore different walkers densities, are considered. Numerical analyses of the transient response to the moving harmonic load were carried out, and the peak acceleration was evaluated. The probability distribution of the peak acceleration induced by the crossing of walkers belonging to the Standard Population is evaluated through Monte Carlo simulations. Finally, the empirical probability distributions are fitted to a Generalized Extreme Value (GEV) distribution and its characteristic parameters are discussed with reference to six case-study footbridges.

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Keywords: Probabilistic assessment; Footbridges; Peak acceleration; Human-induced vibrations.

1. Introduction

Recently-built footbridges are characterized by low stiffness and damping and are therefore prone to human-induced vibrations. Ricciardelli and Demartino[1] compared background hypotheses, field of applicability and results obtained through a number of loading and response evaluation models, concluding that a critical revision of design procedures would be beneficial as these, even though inspired by the same principles and applying the same rules, bring in fact to rather different results [2].

In this context, this paper presents criteria for the probabilistic vibration serviceability assessment of footbridges subjected to single and multiple crossings. The load induced by a single walker is modeled as a moving harmonic force

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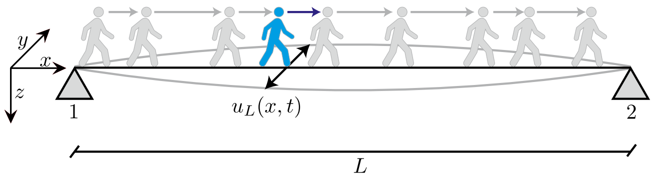


Fig. 1: Footbridge model with walkers.

whose characteristics are derived from a Standard Population (SP) of walkers based on data available in the literature. For multiple walkers, superposition principle is applied. Numerical analyses of transient response to moving lateral harmonic loads are presented, from which the peak response is evaluated in a probabilistic way. In particular, the probability distribution of the peak acceleration induced by the SP is evaluated through Monte Carlo simulations, and the results are fitted to a Generalized Extreme Value (GEV) distribution.

2. Single and multiple walkers vibration response

A common approach for the modeling of dynamic forces induced by a walker is to neglect intra-subject variability, therefore assuming that the force is periodic in time (i.e. that the walker generates identical footfalls with constant frequency). The first harmonic of the lateral component of such periodic force can be expressed as:

$$F(t) = DLF \cdot W \cdot (\sin(\pi\,(f_w/2)\,t) - \psi) \tag{1}$$

where W is the weight of the walker, DLF is the Dynamic Load Factor, i.e. the harmonic load amplitude normalized by the body weight, f_w is the step or walking frequency, and ψ is the phase lag. This expression of the load is used by many Standards, such as UK Annex to EC1 [3] and ISO 10137 [4]. When the walker crosses a footbridge of span L, the first lateral modal load is:

$$f(t) = \int_0^L \phi(x) \cdot F(t) \cdot \delta(x - v \cdot t) \cdot \left[H(t) - H(t - T_p)\right] \mathrm{d}x \tag{2}$$

where $\phi(x)$ is first lateral mode shape, $\delta(\bullet)$ is the Dirac Function, x defines the position of the walker on the bridge, $H(\bullet)$ is the Heaviside function, $T_p = L/v$ is the crossing time being $v = f_w \cdot l_w$ the walking speed with l_w the step length.

For a supported beam the lateral deflection of the footbridge is $u_L(x,t) = \sin(\pi x/L)\cdot\eta(t)$, where $\eta(t)$ is the associated generalized coordinate. The modal equation of motion is:

$$\ddot{\eta}(t) + 4\pi\xi f\dot{\eta}(t) + 4\pi^2 f^2\eta(t) = m^{-1} f(t) \tag{3}$$

where ξ is the modal damping ratio, f is the natural frequency and m is the modal mass. Solving Eq. 3, the response induced by a single pedestrian can be computed.

When a footbridge is crossed by more walkers, a different description of the load must be adopted. In this study, free walking conditions are assumed, i.e. the pedestrian flow is modeled as the superposition of independent walkers with no sensorial feedback. The characteristics of each walker are described on a probabilistic basis and the response is evaluated in the time domain through Monte Carlo simulations, applying the superimposition principle. The modal response is evaluated by summation of the responses associated with each walker with arrival time τ_k, considered as a compound Poisson process, and can be expressed as follows:

$$\eta_N(t) = \sum_{k=1}^{N(t)} \eta_k(t - \tau_k) \quad ; \quad \tau_k = \sum_{\bar{k}=1}^{k} \tau_{0,\bar{k}} \tag{4}$$

where $N(t)$ is a point process [5] and $\tau_{0,\bar{k}}$ is the time-lag between the arrival of the $\bar{k}-$th and $(\bar{k}-1)$th walker, distributed according to $p(\tau_0) = \exp(-\lambda t)$, with λ mean arrival time. The latter can be expressed as a function of the average number N of walkers simultaneously occupying the footbridge, or of the walker density $\delta = N/A$, A being the deck area, as:

$$\lambda = \frac{N \cdot \bar{v}}{L} = \delta \cdot \bar{v} \cdot B \tag{5}$$

where $\bar{v}$ is the mean walking speed of the SP and B is the deck width. The use of this model is bound by a capacity density δ_c, which is the upper limit of free walking conditions. Above this threshold, walking speed decreases with density, the correlation of walking-induced forces increases and interaction among walkers cannot be neglected [1]. The literature does not provide a definitive value of δ_c; reasonable values appear to be 0.2 walkers/m^2 [6], 0.2 – 0.5 walkers/m^2 [7,8], 0.3 – 0.6 walkers/m^2 [9]. In the following, it will be assumed $\delta_c = 0.5$ walker/m^2.

2.1. Standard Population of walkers

The definition of a Standard Population of walkers is needed to characterize inter-subject variability probabilistically. The parameters governing the excitation generated by a walker in the lateral direction are (*i*) the walking speed v, (*ii*) the step frequency f_w, (*iii*) the Dynamic Load Factor *DLF*, (*iv*) the weight of the walker W and (*v*) the phase angle ψ. All these variables are described by Normal distributions, the parameters of which are given in Table 1, and whose Probability Density Functions (PDFs) and Cumulative Distribution Functions (CDFs) are shown in Figure 2. Negative values of the walking parameters are meaningless, and the distributions are truncated at zero. In particular, for the distribution of the walking speed truncation was applied for 0.41 m/s as smaller values lead to negative values of the STD of f_w. The mean and STD of the step frequency are linearly dependent on the walking speed [7]; the mean value of f_w associated with the mean walking speed is 1.898 Hz, and the STD is 0.086 Hz. The *DLF* has a mean value of 0.03792 and STD equal to 0.01459 according to [7]. The walker weight has a mean equal to 744 N and STD equal to 130 N [10]. Finally, phase lags ψ are considered as uniformly distributed between 0 and 2π.

Table 1: Standard Population of walkers: parameters of the Normal distributions and references.

Parameter	Probability distribution	Unit	References
v	$\mathcal{N}(1.41, 0.224) > 0.41$	m/s	Ricciardelli et al.[11]
f_w	$\mathcal{N}(0.7868 \cdot v + 0.7886, 0.0857 \cdot v - 0.035) > 0$	Hz	Butz et al.[7]
DLF	$\mathcal{N}(0.03792, 0.01459) > 0$	–	Butz et al.[7]
W	$\mathcal{N}(744, 130)$	N	Heinemeyer et al.[10]

3. Probability distribution of the footbridge peak acceleration

The peak modal acceleration of the footbridge can be expressed in terms of a TFRF [1,12]:

$$\varphi(\alpha, L, \xi, \delta) = \hat{\ddot{\eta}}(\alpha, L, \xi, \delta) \cdot \left(\frac{\overline{DLF} \cdot \overline{W}}{2\xi m} \right)^{-1} \tag{6}$$

where $\alpha = \bar{f}_w / f$ is the frequency ratio, $\bar{f}_w$ being the mean step frequency; $\hat{\ddot{\eta}}$ is the peak acceleration, $\overline{DLF}$ and $\overline{W}$ are the mean values of the *DLF* and of W within the SP, respectively. The TFRF is the ratio of the peak acceleration induced by a walker crossing the footbridge to the amplitude of the stationary acceleration induced by a walker having mean parameters from the SP, located at midspan. In the case of single walkers, it is a measure of the non-stationarity of the response, whereas in the case of multiple walkers it also accounts for the partial correlation.

The PDF of the peak acceleration expressed in terms of TFRF was estimated through Monte Carlo simulations. The empirical distributions of the TFRF were fitted to a GEV distribution:

$$P_\varphi(\varphi(\alpha, L, \xi, \delta)) = \exp\left\{ - \left[1 + k \left(\frac{\varphi - \mu}{\sigma} \right) \right]^{-(1/k)} \right\} \tag{7}$$

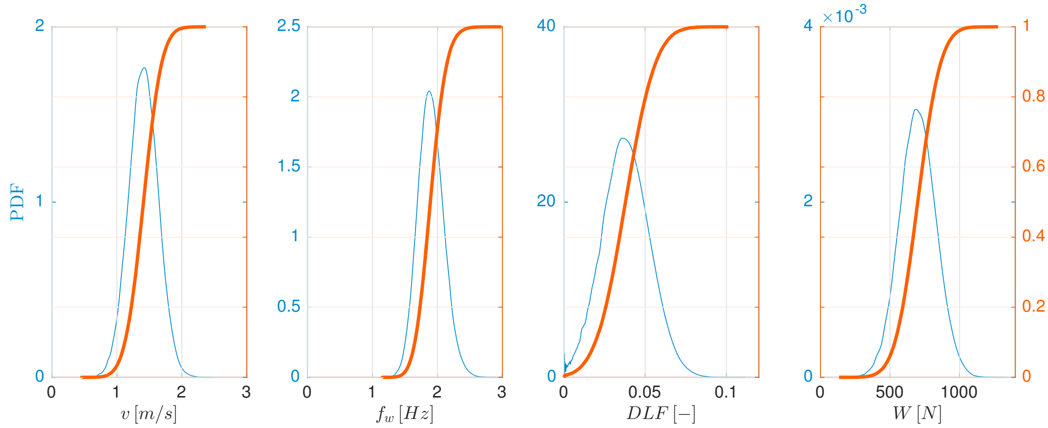


Fig. 2: Probability Density Functions (thin line) and Cumulated Distribution Functions (thick line) of the parameters defining the SP: v, f_w, DLF and W.

Table 2: Analyzed footbridges characteristics and derived peak factors.

Footbridge #	L [m]	B [m]	ξ [%]	ε [-]	α [-]	f [Hz]	K_μ [-]	$K_{\varphi\|_{P_{\varphi_i}=0.95}}$ [-]
1	50	2.5	0.7%	≈ 0.25	0.8; 1; 1.2	1.18; 0.95; 0.84	2.40; 6.31; 4.06	3.27; 8.03; 5.33
2	100		1.4%	≈ 2			4.83; 12.65; 8.29	6.46; 15.86; 10.61

where the location parameter, μ, the scale parameter, σ, and the shape parameter k are all dependent on α, L, ξ and δ.

In the case of a single walker, the following procedure was adopted: a) random generation of N_s walkers, whose characteristics (W, DLF, v and f_w) follow the PDFs of the SP (Section 2.1); b) evaluation of the modal lateral acceleration time histories for each walker, $\ddot{\eta}(t)$; c) evaluation of the lateral peak accelerations in terms of $\varphi(\alpha, L, \xi)$ (Eq. 6); d) evaluation of the empirical PDF of the TFRF, $p_\varphi(\varphi(\alpha, L, \xi))$; e) fit of the empirical PDF to a GEV distribution (Eq. 7 neglecting δ).

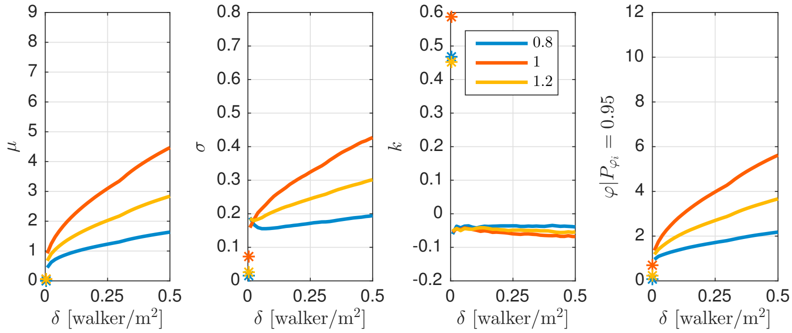
In the case of multiple walkers, for each value of the walkers density the following procedure was adopted: a) random generation of N_s, 3600 s long time histories of the load induced by multiple walkers form the SP with Poisson-distributed arrival time; b) evaluation of the modal lateral acceleration time histories for each time window, $\ddot{\eta}(t)$; c) evaluation of the lateral peak accelerations in terms of $\varphi(\alpha, L, \xi, \delta)$ (Eq. 6); d) evaluation of the empirical PDF of the TFRF, $p_\varphi(\varphi(\alpha, L, \xi, \delta))$; e) fit of the empirical PDF to a GEV distribution (Eq. 7).

4. Case study

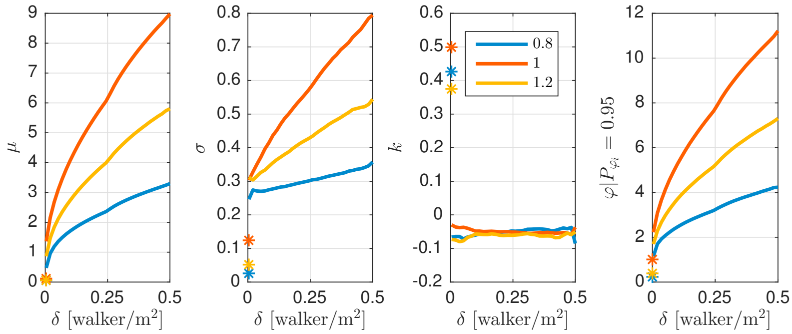
As an example, the proposed procedure is applied to six case-study footbridges. The characteristics of the footbridges were chosen such to cover the typical range of the stationarity parameter $\varepsilon = (2l_w/L)\,\xi$ as reported by Ricciardelli and Demartino[1]. These are the combination of two span lengths and three vibration frequencies, as reported in Table 2.

Monte Carlo simulations were carried out to characterize the probability distribution of the footbridge peak acceleration for single and multiple crossings, using the procedure reported in Section 3. In particular, it was set $N_s = 1{,}000$, satisfying the requirement of stability of the solution (also comparing some results with $N_s = 10{,}000$). In the case of multiple crossings, the walker density was varied from that corresponding to a single pedestrian (varying with deck area), to the capacity density $\delta_c = 0.5\,\text{walker/m}^2$.

The estimated GEV parameters for the single walker case and for the multiple walkers case are shown in Figure 3 together with the value of the TFRF corresponding to a cumulated probability of 0.95, $\varphi|_{P_\varphi=0.95}$ (Eq. 7). The modal masses of the analyzed footbridges are not reported since according to Eq. 6 the peak acceleration can be calculated for any value of the modal mass.



(a) Footbridge #1.



(b) Footbridge #2.

Fig. 3: GEV parameters for the single walker, $P_\varphi(\varphi(\alpha, L, \xi))$, for α = 0.8, 1 and 1.2 (asterisk markers) and for the multiple walkers, $P_\varphi(\varphi(\alpha, L, \xi, \delta))$, as a function of δ for α = 0.8, 1 and 1.2 (solid lines). $\varphi|_{P_\varphi=0.95}$ indicates the value of the TFRF corresponding to a cumulated probability of 95%.

Looking at the single crossing walkers case (asterisk markers in Figure 3), σ broadly range from 0.02 to 0.08 for footbridge #1 and from 0.03 to 0.145 for footbridge #2, taking larger values at resonance. Moreover, larger values of σ were evaluated for $\alpha = 1.2$ compared with $\alpha = 0.8$. μ broadly range from 0.01 to 0.07 for footbridge #1 and from 0.03 to 0.12 for the footbridge #2 with a similar distribution of σ. The increase of σ and μ from footbridge #1 to footbridge #2 is due to the increase in the stationarity parameter. As observed from the results, the increase in damping is not sufficient to counterbalance this effect. k ranges from 0.35 to 0.6, taking the largest values at resonance, which indicates a Fréchet or Type II Extreme Value behavior. For footbridge #1, only slightly larger values of k were found compared to those of footbridge #2, indicating a similar probabilistic behavior of the extreme response.

Looking at the multiple crossings case (solid lines in Figure 3), it is observed that μ increases approximately with the square root of the walker density, $\sqrt{\delta}$. This is consistent with the results of Matsumoto et al.[13], who noted that when a footbridge is crossed by a stream of walkers having equal frequency and phases uniformly distributed between 0 and 2π (uncorrelated walkers), the RMS response varies with the square root of δ. Accordingly, it can be written:

$$\mu(\alpha, L, \xi, \delta) = K_\mu(\alpha, L, \xi) \cdot \sqrt{\delta} \tag{8}$$

The values of K_μ for the six analyzed cases are reported in Table 2. Consistently with the results reported in Figure 3, K_μ increases at resonance and takes larger values for footbridge #2. However, it exhibits a different trend compared with μ. In particular, at resonance, it has an almost linear behavior in the μ vs. $\sqrt{\delta}$ plane, whereas away from resonance it appears linear in the μ vs. δ plane. Both σ and k show an irregular behavior for very low values of δ; this is ascribed to the low number of crossings and to the low number of load cycles over which the peak is estimated, leading to larger uncertainties in the statistical estimation. For larger values of δ, k stabilizes around an almost constant value of -0.05 indicating a Gumbel or Type I Extreme value behavior (i.e. $k = 0$).

Using the estimated GEV parameters, the TFRF corresponding to a cumulated probability of 95% was also evaluated; the latter value was chosen being the characteristic fractile commonly adopted in Civil Engineering applications for the definition of the demand parameters. Considering the single walker case, $\varphi|_{P_\varphi=0.95}$ is 0.66 and 0.99 at resonance for footbridges #1 and #2, respectively. This indicates that in the footbridge #2 the peak response is almost equal to the stationary response induced by the mean walker from the SP. On the other hand, in non-resonant conditions, $\varphi|_{P_\varphi=0.95}$ takes low values of 0.11 ($\alpha = 0.8$) and 0.21 ($\alpha = 1.2$) for footbridge #1 and of 0.20 ($\alpha = 0.8$) and 0.36 ($\alpha = 1.2$) for footbridge #2. Considering the case of multiple walkers, it is observed that, similarly to μ, $\varphi|_{P_\varphi=0.95}$ increases approximately with the square root of the walker density, taking larger values at resonance and for footbridge #2. Also in this case it can be written:

$$\varphi|_{P_\varphi=0.95}(\alpha, L, \xi, \delta) = K_{\varphi|_{P_\varphi=0.95}}(\alpha, L, \xi) \cdot \sqrt{\delta} \tag{9}$$

The values of $K_{\varphi|_{P_\varphi=0.95}}$ estimated for the six analyzed cases are reported in Table 2.

5. Conclusions

From the results of this study, the following conclusions can be drawn:

- The probability distribution of the footbridge peak acceleration to a single crossing walker is a Fréchet or Type II Extreme Value Distribution.
- The probability distribution of the footbridge peak acceleration to multiple crossing walkers is a Gumbel or Type I Extreme Value Distribution whose mean, μ, increases approximately with the square root of the walker density.
- For the six analyzed footbridges, the slopes of μ and $\varphi|_{P_\varphi=0.95}$ with respect to $\sqrt{\delta}$ have been derived.
- The procedure can be used to evaluate the variation of μ and $\varphi|_{P_\varphi=0.95}$ with respect to $\sqrt{\delta}$ for any footbridge and for any fractile, thus making a complete probabilistic model of the acceleration demand induced by multiple walkers in free walking conditions, ready for implementation in Standards and Code of Practice.

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