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Dynamic control of water distribution system based on network partitioning

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Abstract

The availability on the market of remote control valves for water distribution systems allows a more flexible implementation of the "divide and conquer" paradigm, that consists in dividing large networks into smaller district meter areas defining a water network partitioning (WNP), aiming at controlling water balance, pressure levels and water quality protection. The positioning of gate valves is carried out using optimization approaches to guarantee the network reliability that can be significantly reduced by WNP owing to the closure of several pipes by means of gate valves, decreasing topologic and energy redundancy. Anyway, starting from the optimal positioning of remote controlled gate valves, obtained with SWANP software, the paper investigates the effectiveness of dynamic control, in order to face hydraulic failure in fire estinguishment. The proposed methodology, based on heuristic optimization algorithm, finds the optimal layouts minimizing the number of valves to be opened and maximizing the system performance. The study highlights the advantages of adaptively reconfigurable networks starting from a partitioned system, confirming that a dynamic control represents a significant improvement for smart water networks.

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1. Introduction

The water distribution network (WDN) complexity, often with thousands of nodes and pipes and length of hundreds of kilometres, is the main reason for an arduous management uncapable of preventing water leakage and planning optimization actions [1]. Recently, the paradigm of *divide and conquer* [2] has been proposed, to define a water network partitioning (WNP) to reduce the network complexity in different smaller permanent District Meter Areas (DMA), in which controlling water balance, pressure levels and water quality protection is easier [2, 3].

Water network partitioning is achieved by positioning flow meters and gate (or cut off) valves along the pipes, using either empirical or optimization approaches to guarantee the network performance, defining permanent DMA with a variable number of users between 2,000-3,000 from 10,000-15,000 [3, 4]. The design of some permanent DMA offers: a) a good solution to define portions of network in which monitoring inflow and outflow by flow meters allows controlling water budget to identify where water losses occur [1]; b) a smart technique to choose the optimal locations for Pressure Reduction Valves (PRV), to achieve pressure management and reduce leakage [5]; c) and, finally, an effective way to protect the network from accidental and malicious contamination, by sectorizing polluted DMA [6]. Nevertheless, water network partitioning opposes to the traditional approach to design water distribution system [7] with high degree of redundancy both in terms of topology (many loops) and energy (pipe diameters larger than required). Indeed, inserting several gate valves along pipes can reduce drastically water performance in terms of topologic redundancy and energy resilience, thus decreasing the robustness and resilience of the water distribution system [8].

For this reason, only recently, overcoming empirical approaches [8, 9], different techniques have been proposed in scientific literature based on network analysis, graph and network theory integrated in optimization methods [10].

Hydraulic simulations to achieve the optimal WNP are carried out with respect to peak water demand, that usually represents the ordinary operation condition more challenging in terms of hydraulic performance [11]. Anyway, water distribution network can encounter other extraordinary conditions, such as fire, pipe breaks, insufficient pumping or storage capacity, unplanned water demand peaks that, for a partitioned system, can be disastrous in terms of performance not guaranteeing the level of service for fire hydrants or for users. Furthermore, these situations can occur more frequently in real water distribution systems with low original hydraulic performance in terms of resilience and robustness, depending on topologic and hydraulic characteristics.

The availability on the market of remote controlled gate valves can be an effective solution, transforming a static network layout, that can get into troubles, in a dynamic layout, that can allow to overcome different problems. In other terms, starting from the positioning of gate valves using an optimization approach to reduce the decrease of resilience, it is possible to identify some dynamic layouts, opening some gate valves, that ensure the fulfilment of different levels of service.

Only recently, an adaptive approach has been proposed by Wright et al. [12] to overcome the drawbacks of the DMA design that reduces redundancy in network connectivity which has a severe impact on network resilience, incident management and water quality deterioration. The approach integrates the benefits of DMAs for leakage management with the advantages of large looped networks for increased redundancy in connectivity, reliability and resilience. However, the proposed methodology, based on sequential convex programming as optimization techniques, is applied to optimize valve settings at all locations and the boundary closing at night in order to define a dynamic topology reconfiguration and pressure control for water distribution networks. Wright et al. [12] inserted in the water system network controllers, designed and integrated with novel telemetry tools for high-speed time-synchronized monitoring of the dynamic hydraulic conditions, bringing traditional static water distribution management closer to the current driver for smarter water networks [12]. A linear programming approach was used also by Perelman et al. [13] for a flexible reconfiguration of existing water distribution infrastructure, which is adaptive to the water utility constraints. The network layout is reconfigured, based on graph theory, defining a star-like topology, where the center node is a connected subset of transmission mains, that provides connection to water sources, and the nodes are the subsystems that are connected to the sources through the center node.

In this paper, starting from an optimal WNP layout designed with the need of SWANP software [14], the effectiveness of dynamic control by remote-controlled valves, in order to face hydraulic failure during fire estinguishment, finding the optimal layouts of boundary valves, minimizing the number of valves to be opened and maximizing system performance, have benne investigated. The proposed case study confirms that, starting from a layout optimized for the aims of water network partitioning, dynamic control represents a significant improvement towards the new management possibilities offered by the smart water network paradigm.

2. Water network partitioning

The paradigm of *divide and conquer* in WDN consists of clustering the water network nodes into groups according to assigned constraints (i.e., density, balancing, boundary edge minimisation, size limit, etc.).

The procedures are generally subdivided in two phases: 1) the clustering, aimed at defining the shape and the dimension of the network subsets based on different procedures aimed at minimizing the edge-cut number N_{ec} and balance the number of nodes for each district, using graph algorithms [15], [16], [17], [18], multilevel partitioning [19], community structure [20], and spectral approach [21], 2) the physical partitioning, that is the selection of the pipes where flow meters or gate valves have to be installed, based on iterative [17] or optimization algorithms [18], with the objective of defining the optimal layout that minimizes the cost and the hydraulics deterioration [19]. Both phases are very complex in terms of computational burden.

Specifically for the first phase, let's consider the water network as a simple weighted graph G = (V, E), where V is the set of n vertices (or nodes) and E is the set of m edges (or links) and denote as ε_{ij} the non-negative weight of the edge $ij \in E$ and $\varepsilon_{ij} = 0$ if $ij \notin E$. Let ω_i be a positive weight of vertex $i \in V$ and $\omega(D) = \sum_{i \in D} \omega_i$, where $D \subseteq V$.

The graph partitioning problem consists of dividing V vertices of G into k subsets, $D_1, D_2, ..., D_k$, such that $D_i \cap D_j = \emptyset$ for $i \neq j$, and $D_i = n_i$ with i = n minimizing the deviation $\binom{n_i}{n_i} \binom{n/k}{n_i}$ and the number of the edges-cut $N_{ec} = \sum_{i \in D_n \Rightarrow j \notin D_n} e_{ij}$ (with e_{ij} called the edge-cut) that connect vertices in different partitions.

In the cases in which the vertices and edges have weights associated, the goal is to partition the vertices into k disjoint subsets such that the sum of the vertex-weights in each D_i is the same and the sum of the edge-weights whose incident vertices belong to different subsets is minimized. Then, once obtained the set N_{ec} of the edge-cuts, it is necessary to choose how many and which of these boundary pipes have to be interrupted with N_{gv} gate valves and, at the same time, which ones have to be used for installing N_{fm} =(N_{ec} - N_{gv}) flow meters. In other terms, for assigned k districts, once the possible positions e_{ij} for flow meters and boundary valves have been defined by the graph partitioning technique, and once the numbers N_{fm} and N_{gv} have been chosen, one should define which pipes have to be interrupted among all the possible combinations of WNP layouts N_c expressed by binomial coefficient:

$$N_c = \begin{pmatrix} N_{ec} \\ N_{fm} \end{pmatrix} \tag{1}$$

which, evidently, grows enormously even for a relatively small number of districts. Therefore, also in this case, the problem is practically unsolvable with an exhaustive search of best solution and the recourse to heuristic methods is needed. So, once fixed the number N_{fin} , it is achieved by means of a heuristic procedure, based on a Genetic Algorithm (GA) developed by the authors in [11, 14]. The GA allows the determination of the optimal position of each flow meter in the network by inserting gate valves in the pipes that belong to the *edge-cut* set, minimizing the dissipated power of the water system.

Both phases have been integrated in the software SWANP 3.0 [13], developed in Python v2.7.6 by some of the authors, to obtain, automatically, an optimal partitioning of a water distribution network. SWANP provides to the decision-maker different solutions comparing network layouts with some hydraulic and protection performance indices [23]. Specifically, some algorithms based on graph partitioning (*Multi-Level Recursive Bisection*, MLRB) [22]) and community structures (*Edge Betweenness Community*, EBC) [12]) approaches have been implemented in SWANP allowing to obtain different layouts of network partitioning based on the choice of number of districts, weights on pipes and/or nodes, and a set of performance indices to compare the layouts and choose the best solution.

The case study is the medium-sized network of Giugliano in Campania in the North of Naples in Italy, illustrated in Fig. 1, with 123.000 inhabitants. In Table 1, the main characteristics of the Original Water Network (OWN) are reported, with a 567 nodes and 612 pipes in plastic, iron and concrete. The network is supplied by 5 sources consisting of reservoirs and pump systems. The average demand is $0.223 \, \text{m}^3/\text{s}$ with a peak demand equal to $0.290 \, \text{m}^3/\text{s}$ in compliance with a design pressure $h^*=25 \, \text{m}$. The network model was simplified deleting all secondary pipes with a very low diameter.

The hydraulic simulation of the network was carried out as a Pressure Driven Analysis [24], because this approach is more appropriate for extraordinary service conditions, such as water partitioning, peak demand, fire service, when the water pressure can fall below the design pressure.

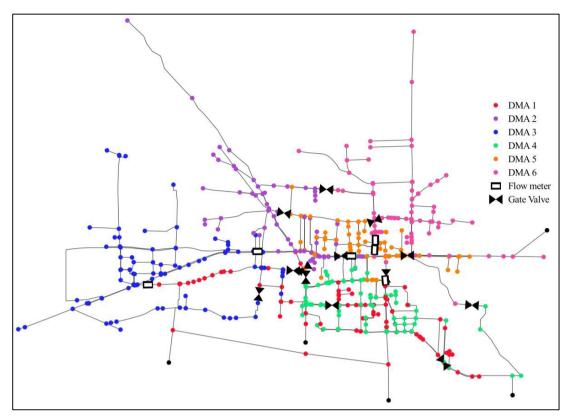


Fig. 1. Parete Partitioned Water Network (PWN) with flow meters and gate valves

The water network of Giugliano in Campania was partitioned by SWANP 3.0 software, through a MLRB algorithm with no weights on pipes and water demands, in 6 DMAs, as illustrated in the Figure 1, highlighting districts with different node colours obtained with 13 gate valves and 6 flow meters.

nodes n [-]	links m [-]	nine length		Pipe materials [-]	Average demand [m³/s]	Peak demand [m³/s]	Design pressure h* [m]
567	612	5	81	Plastic, iron and	0.223	0.290	25

Table 1. Main characteristics of the Giugliano in Campania network in peak water demand condition

In Table 2, some hydraulic and topologic indices are reported both for OWN and for PWN (Partitioned Water Network), specifically, the following indices are computed:

a) energy performance indices (EPI), traditionally expressed by mean node pressure h_{MEAN} , maximum node pressure h_{MAX} , minimum node pressure h_{MIN} and standard deviation node pressure h_{SD} and, more recently, by the resilience index I_R [25], i.e. the ratio between the power dissipated in the network to satisfy the total demand and the maximum power that would be dissipated internally in order to satisfy the constraints in terms of demand and head at the nodes, and by the resilience deviation index I_{RD} [11], based on the comparison among the resilience indices of the original and partitioned network;

b) topological performance indices (TPI), measured by the Meshedness Coefficient [26], introduced to estimate the redundancy of the network layout, computed by the following equation:

$$C_m = \frac{m - n - 1}{2n - 1} \tag{2}$$

As highlighted in Table 2, the OWN has a low value of C_m =0.0039 index that measures the network topologic redundancy, showing a network structure as a "tree network", but has a high value of resilience index I_r =0.884, that measures the energy redundancy with a design pressure equal to 25 m in the peak water demand (with 1.3 peak coefficient). This energy redundancy is due to the presence of some pressure groups that pump up to 170 m.

Table 2. Hydraulic and topologic performance of Giugliano in Campania network (OWN and PWN) in peak water demand condition

	h_{MIN} [m]	$h_{MEAN}[m]$	$h_{MAX}[m]$	$h_{SD}[m]$	$I_R[-]$	$I_{RD}[\%]$	C_m
OWN	55.42	66.49	118.28	4.96	0.884	-	0.039
PWN	52.28	64.04	116.68	6.07	0.831	5.99	0.027

A good compromise, in peak water demand, is achieved with SWANP 3.0 software, with a resilience deviation index I_{RD} =5.99% as reported in Table 2. The PWN is obtained, as reported in Table 3, inserting 6 flow meters and 13 remote-controlled valves (R-C valves) that close the same number of pipes. All *EPI* are good with a slight decrease of pressure and energy resilience in the PWN in compliance with the level of service in peak water demand.

3. Dynamic layouts in fire conditions

As expected, in fire conditions computed as suggested by the Italian law (to insert a demand fire flow Q_f *=30 l/s in each node, one at once, guaranteeing a pressure h_f * at least equal to 15 m and Q_r =0.8Q), the water network partitioning does not guarantee the level of service. Specifically, hydraulic simulation in fire conditions were carried out evaluating all nodes in which the flow and pressure resulted lower than those required by the Italian law.

To overcome this drawback, a dynamic topology was designed analyzing which and how many gate valves have to be motorized and remote controlled in order to allow the fulfillment of the level of service in the fire conditions. The analysis is carried out with a novel approach that considers the water network as a dynamic system with a topology that can be changed opening or closing remote controlled valves in the network.

Anyway, the problem to achieve the optimal layout with the minimization of the number of remote controlled valves is complex because, similarly to equation (1), the number of possible combinations can be huge and a combinatorial analysis is impossible. For this reason, a heuristic optimization procedure, based on the Genetic Algorithm [11, 14], is adopted to find dynamic layouts defining which R-C valves have to be opened in order to find an optimal solution to guarantee the level of service in fire conditions. In other terms, once found the optimal water network partitioning, it is possible to improve the level of pressure in some nodes finding other Dynamic Water Network (DWN) layouts, that satisfy the fire constraints in terms of node flow and pressure, reducing the number of R-C valves closed. This goal can be achieved evaluating all the possible dynamic combinations N_{dc} , again given by the binomial coefficient:

$$N_{dc} = \begin{pmatrix} N_{gv} \\ N_{open} \end{pmatrix} \tag{3}$$

with N_{open} is the number of R-C valves opened.

Note that, for a large number of gate valves N_{gv} , the number N_{dc} is huge and the required computation time is very high and a heuristic optimization procedure is required.

In this work, the GA [11, 14] was used to minimize the following Objective Function:

$$FO = \sum_{i=1}^{n} \alpha_{i} \begin{cases} h_{f,i} > h_{f^{*}} \Rightarrow \alpha_{i} = 0 \\ h_{f,i} < h_{f^{*}} \Rightarrow \alpha_{i} = 1 \end{cases}$$

$$(4)$$

where $h_{f,i}$ is the pressure in fire condition at *i-th* node, finding the determination of R-C valves opening in the network.

In Table 3, simulation results are reported for a comparison between different layouts in fire conditions using the following performance statistical indices, computed in PDA:

$$h_{f,\min} = \min \left\{ h_{f,i} \right\} \text{ with } i = 1, \dots, n \tag{5}$$

$$h_{f,mean} = \sum_{i=1}^{n} h_{f,i} / n \tag{6}$$

$$h_{f,\max} = \max \left\{ h_{f,i} \right\} \text{ with } i = 1,\dots,n \tag{7}$$

$$FDI_{f,\min} = \min \left\{ FDI_{f,i} \right\} \text{ with } i = 1,...,n$$
(8)

$$FDI_{f,mean} = \sum_{i=1}^{n} FDI_{f,i} / n \tag{9}$$

$$FDI_{f,\max} = \max \left\{ FDI_{f,i} \right\} \text{ with } i = 1,...,n$$
 (10)

where $FDI_{f,i}$ is the Flow Deficit Index in the whole network, computed as follows:

$$FDI_{f,i} = 1 - \sum_{j=1}^{n} Q_{j,a} / \sum_{j=1}^{n} Q_{j,r} \begin{cases} j = i \Rightarrow Q_{j,r} = Q_{f^*} \\ j \neq i \Rightarrow Q_{j,r} = 0.8 \cdot Q \end{cases}$$

$$(11)$$

where $Q_{j,a}$ is the actual demand delivered in PDA approach and $Q_{j,r}$ is required demand equal to Q_f^* in the node in which occurs the fire and $Q_{j,r}$ in all other nodes.

In the first line of Table 3, the results for the original network (OWN) show that in the fire conditions all network nodes satisfy flow and pressure condition with $h_{f,MIN}$ =19.04 m (well above 15 m) and all $FDI_{f,MIN}$ =0 (without flow deficit) and, consequently, also $FDI_{f,MEAN}$ =0 and $FDI_{f,MAX}$ =0. Conversely, the results for the partitioned water network (PWN) are significantly worse in fire conditions, with $h_{f,MIN}$ =1.98 m and $FDI_{f,MIN}$ =0 (without flow deficit in 496 nodes) but $FDI_{f,MEAN}$ =0.42 and $FDI_{f,MAX}$ =8.10, that highlight flow deficit in 71 nodes with $h_f < h_{f^*}$ and $Q_f < Q_{f^*}$.

Table 3. Comparison of different layout in fire condition

	R-C	Flow	R-C ON	Nodes	Nodes	$h_{f,MIN}$	$h_{f,MEAN}$	$h_{f,MAX}$	$FDI_{f,MIN}$	$FDI_{f,MEAN}$	$FDI_{f,MAX}$
	valves	meters	valves	$h_f < h_{f^*}$	$Q_f < Q_{f^*}$	[m]	[m]	[m]	[%]	[%]	[%]
OWN	-	-	-			19.04	68.90	119.35	0	0	0
PWN	13	6	-	71	71	1.98	67.53	118.74	0	0.42	8.10
DWN 1	13	6	1	49	49	7.10	67.51	118.73	0	0.19	4.62
DWN 2	13	6	2	32	32	7.10	67.54	118.73	0	0.14	4.62
DWN 3	13	6	3	15	15	7.10	67.59	118.73	0	0.05	3.70
DWN 4	13	6	4	7	7	11.13	67.63	118.74	0	0.02	1.99
DWN 5	13	6	5	-	-	17.95	67.69	118.78	0	0	0

From the 3rd to 5th line of Table 3, optimal simulation results are reported for five DWN layouts obtained opening respectively, 1, 2, 3, 4 and five R-C valves. In particular, opening only one R-C valve, as indicated in the 3th column of Table and illustrated in the Figure 2, it is possible to reduce, from 71 to 49, the number of nodes in which the pressure and the flow are lower than required from the Italian law for fire conditions and improve all Flow Deficit Index from 0.42 to 0.19. This result, found by GA, is very useful because if the fire occurs in one of the 71-49=22 red nodes illustrated in the Figure 2, the water system managers know which valve has to be opened to provide the required water to extinguish the fire.

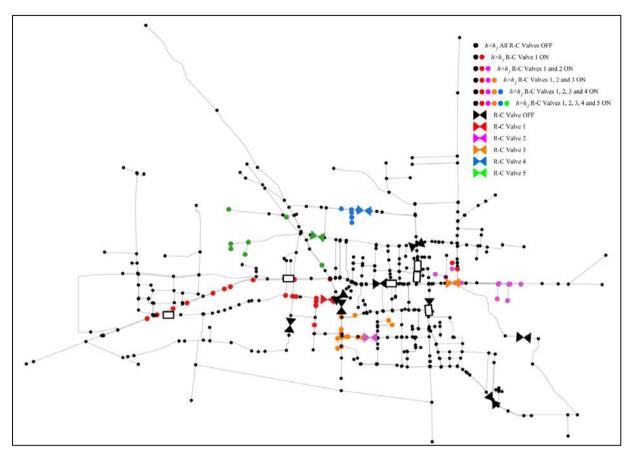


Fig. 2. Dynamic Water Network layouts in fire condition.

With the heuristic optimization algorithm, it is possible to find the optimal dynamic layouts from one to the maximum number of contemporary R-C valve ON opened, that guarantee the required level of service in each network nodes, that in the Giugliano network, is equal to five, as illustrated in Table 3 and Figure 2. Of course, in the case reported in the last line of Table 3, with 5 R-C valve ON and 8 R-C valve OFF, no nodes have pressure lower than fire pressure (15 m) and fire flow (30 l/s) with h_{6MIN} =17.95m; h_{6MEAN} =67.69m and h_{6MAX} =118.78m.

4. Conclusions

The paper proposes a methodology to define dynamic layouts, in fire estinguishment conditions, of water distribution network of Giugliano in Campania (Italy), previously partitioned in 6 DMA with several gate valves and some flow meters to achieve the advantage of water network partitioning. Dynamic layouts consist in different network configurations – obtained with a heuristic optimization algorithm, based on genetic algorithm – achieved opening one or more remote controlled valves. This approach allows to change the traditional management of water distribution networks, essentially based on a static topologic layout. Indeed, the availability of low cost remote control devices allows to re-design network topology, starting from existing real water systems, to improve the ordinary and extraordinary operation conditions, network partitioning can be very effective to compute the water balance in each district meter area and, consequently, to reduce the water losses, or to achieve pressure management and water quality protection, but it can worsen significantly the hydraulic performance of the network in the case of concentrated peak demand, such as in fire estinguishment conditions. The definition of dynamic layouts represents a

good way to overcome this drawbacks, satisfying the level of service in fire conditions. In this way, it is possible also to choose which gate (or boundary) valves have to be remote controlled, minimizing the investment costs.

Anyway, dynamic layouts can face also other issues in extraordinary operation conditions, such as pipe breaks, insufficient pumping, unplanned storage capacity, unusual water demand peaks that, for a partitioned system, can represent drastic conditions. Therefore, the authors are studying other applications of the proposed novel approach.

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